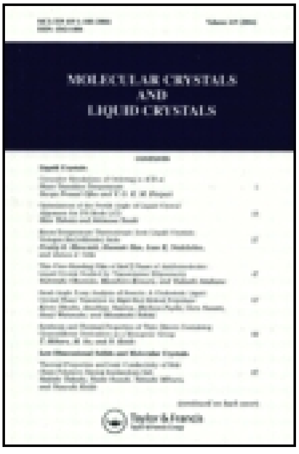

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DISCLINATION BEHAVIOUR IN THE FLOW OF NEMATIC LIQUID CRYSTAL

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Abstract The problem of disclinations streamlining by the flow of nematic liquid crystal (NLC) was investigated theoretically. Distributions of the flow velocity for defects with different topological indices were found numerically.

INTRODUCTION

The hydrodynamic motion of nematics for simple macroscopical flow situations as well as for small deviations of the director about the completely oriented state have been studied well enough.[1,2] However, the formation of NLC is nearly always accompanied by disclinations (both linear and point) and the problem of defect behaviour in the nematic flow is of great interest.

THEORETICAL RESULTS AND DISCUSSION

To study this problem it is necessary to solve nematodynamics equations for the initial director field distribution with disclinations. There are certain difficulties here due to the fact that with defect motion both the director $\vec{n}$ and the flow velocity $\vec{v}$ are time and coordinate functions, so that the resulting system of nonlinear differential equations for $\vec{n}$ and $\vec{v}$ cannot be solved even for a case of one disclination in the nematic flow, to say nothing of defect pairs.

Recently, the gauge fields (or vector potentials) have been introduced to describe the hydrodynamics of disclinations in NLC within the small and large disclination density limits.[3,4] However, the use of gauge transformations requires the construction of a new system of nematodynamics equations and the relevance of the latter needs further experimental verification.

Within the frame of Ericksen-Leslie nematodynamics theory[1,2], the following approach is suggested in this paper: if the velocity of NLC flow considerablly exceeds that of the disclination motion the defects are meant to be fixed, so that at stationary flow we have $\vec{n}=\vec{n}(\vec{z})$, $\vec{v}=\vec{v}(\vec{z})$. It is not difficult to calculate the flow velocity and the director distributions for different cases of disclination configurations.

The orientation of the director and the flow velocity distribution are obtained generally from the system of nematodynamics equations[1,2]

$$\begin{cases} J\dfrac{d}{dt}\left(\vec{n}\times\dfrac{d\vec{n}}{dt}\right)=(\vec{n}\times\vec{h})-(\vec{n}\times\vec{R})\,, \\ \rho\dfrac{dv_i}{dt}=\Sigma_{ji,j}\ ; \end{cases} \qquad (1)$$

Here and further the comma denotes a partial derivative over the spacial coordinates ($q_{i,j}\equiv\partial q_i/\partial x_j$); J is the inertia moment of director; $\vec{v}$ is the flow velocity; and $\vec{h}$ is the molecular field making the director to take an equalibrium orientation

$$h_i=\left(\frac{\partial F}{\partial n_{i,j}}\right)_{,j}-\frac{\partial F}{\partial n_i}\ ,$$

where F is the elastic free energy density of NLC in the absence of external fields; $\vec{R}$ is the dissipative force and Σ_{ij} is the "viscous" tensor in NLC.

Consider a stationary plane flow of NLC at the velocity $v \gg v_{disc}$ along OX axis in the Cartesian coordinates. The disclination line oriented along OZ axis we shall take as an initial (undisturbed by the flow) director distribution, then we have $\vec{n} = \vec{n}(x,y)$, $\vec{v} = \vec{v}(x,y)$. Let us linearize the system (1) for a case of one - constant approximation: $n_x = \cos U$, $n_y = \sin U$, $\vec{n}^2 = 1$, $U = U_0 + U_1$, where U_0 is the initial director distribution satisfying to the equilibrium equation $\nabla^2 U_0 = 0$, $U_0 = S\,\mathrm{arctg}(y/x)$, S is the topological index of disclination; U_1 is the disturbance caused by the flow; $\vec{v} = \vec{v}_0 + \vec{v}_1$, where $\vec{v}_0$ is the flow velocity at the enough large distance from disclination; $\vec{v}_0 = (v_0, 0, 0)$; $\vec{v}_1 = (v_{1x}, v_{1y}, 0)$ is the correction to the velocity associated with inhomogenety of director distribution. Besides, in real NLC $Jv_0^2 \ll K$ ($J \sim 10^{-14}$ g/cm, $K \sim 10^{-6}$ din) and the inertia moment of director may be neglected. With dimensionless variables $\xi = x/z_0$, $\eta = y/z_0$, $\vec{V} = \vec{v}/v_0$ where $z_0 = K/\gamma_1 v_0$ denotes the characteristic distance from disclination axis where the director field is not yet disturbed by the flow, we shall have:

$$\begin{cases} U_{1,\xi\xi} - U_{1,\xi} + U_{1,\eta\eta} = U_{0,\xi} V_\xi + U_{0,\eta} V_\eta \\ a_{11} V_\xi + a_{12} V_\eta = a_{13} U_{1,\xi} + a_{14} U_{1,\eta} + a_{15} U_{1,\xi\xi} + a_{16} U_{1,\xi\eta} \\ a_{21} V_\xi + a_{22} V_\eta = a_{23} U_{1,\xi} + a_{24} U_{1,\eta} + a_{25} U_{1,\xi\xi} + a_{26} U_{1,\xi\eta} \end{cases} \quad (2)$$

Here a_{ij} are the functions of U_0 and its partial derivatives over ξ and η. The system (2) was solved numerically by the simple iterative method. The symmetry of the problem ($U_{1,\xi}|_{\xi=0} = 0$, $U_{1,\eta}|_{\eta=0} = 0$) and the condition $\vec{V} \to (1,0,0)$ at $\xi,\eta \to \pm\infty$ have been used. The distributions of the flow velocity for streamlined disclinations with different topological indices are shown in Fig.1 (the maps are plotted according to calculations).

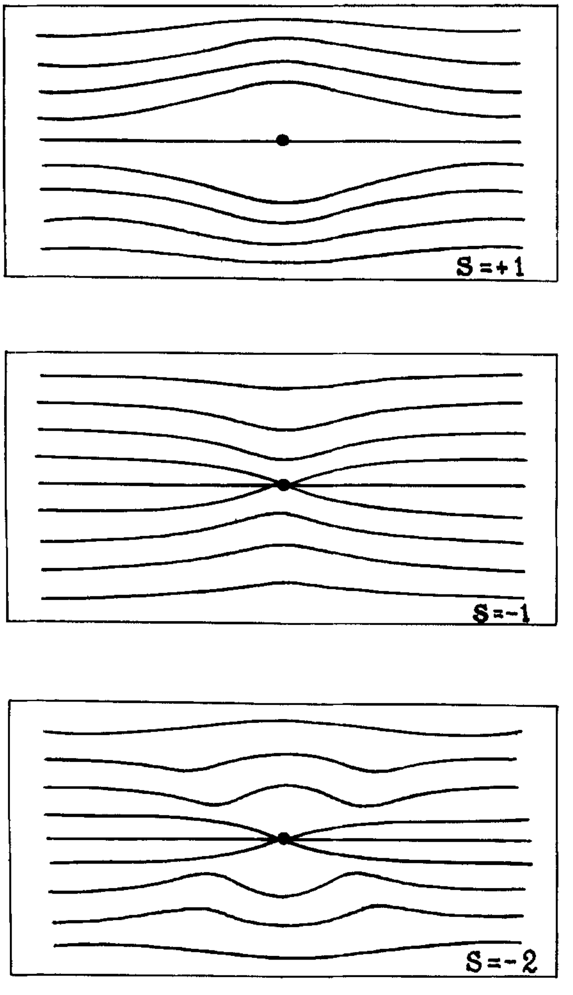

FIGURE 1 The current line distribution in the vicinity of linear disclinations with different topological indices (parallel view to the disclination axis •).

It can be seen that the NLC flow properties depend strongly on the sign of defect: the linear disclination with index S =+1 "repels" the flow (Fig. 1a) whereas disclination with S =-1 "attracts" the flow (Fig. 1b).

Thus, our approach to the problem of disclination behaviour in the NLC flow enables to obtain the streamline patterns of disclinations placed in the flow. It should be noted that in the isotropic liquids the fixed solids play the role of obstacles of the flow. However, in the case anisotropic liquids, the singularities of director field (defect cores) lead to the absence of the flow within them, i.e., the defects themselves play the role of obstacles. Thus, due to interaction of hydrodynamics flow with the order parameter field, a number of interesting phenomena can be realized in the anisotropic liquids, which are absent in the isotropic ones.[4]

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